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Constructive influence of noise-flatness in correlation ratchets

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Abstract. The influence of noise-flatness on overdamped motion of Brownian particles in a 1D periodic system with a simple sawtooth potential subjected to both unbiased thermal noise and three-level telegraph noise is considered. The exact formula for the stationary probability flux (current) is presented. The phenomenon of multiple current reversals and some topological properties of the hypersurface of zero current in the parameter space of noises are investigated and illustrated by phase diagrams. The conditions for the existence of four current reversals versus the switching rate of nonequilibrium noise are given. An alternative interpretation of the results in terms of cross-correlation between two dichotomous noises is presented.

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1 Introduction

Recently, the noise-induced anomalous transport phenomena of Brownian particles in nonlinear periodic structures have been the topic of a number of physical investigations. Among them, we can mention the ratchet effect [1,2], hypersensitive response [3], absolute negative mobility [4], giant amplification of diffusion and noise enhanced stability [5], to name but a few. Active analytical and numerical studies of various models in this field were stimulated by their possible applications in chemical physics, molecular biology, nanotechnology, and for separation techniques of nanoobjects [2,6,7].

The recent boom of the ratchet effect, i.e., directed motion of Brownian particles in spatially periodic structures induced by nonequilibrium fluctuations, with no macroscopic driving applied, started in 1993 with Magnasco's theoretical work [1]. The initial motivation in this field has come from biology, in particular from the studies of the mechanism of vesicle transport inside eukaryotic cells [1,8]. Beyond that, it was suggested that the ratchet mechanism can be used for obtaining efficient separation methods of nanoscale objects, e.g., DNA molecules, proteins, viruses, etc. [2,9,10]. The classification of different types of ratchets (correlation, flashing, etc.) can be found in reference [2]. To date, the feasibility of particle transport by man-made devices has been experimentally demonstrated for several ratchet types [7,10,11].

It should be noted that the dynamics in ratchet structures with its inherent spatial asymmetry generally exhibits a rich complexity, such as the occurrence of multiple current reversals (CRs) and multipeaked current characteristics [2,12]. The models with CRs are potentially very useful, because, compared to the models with no CRs, CRs could lead to more efficient fluctuation-induced separation of particles [2]. In this context, the CRs phenomenon is one of the most interesting aspects of the theory of Brownian ratchets. It has been shown that the CR effect is attainable in various ways, including variations of the flatness parameter of noise [13–15], the correlation time of nonequilibrium fluctuations [16], the temperature in multinoise cases [17,18], the power spectrum of the noise source [19], the shape of the potential [20], the number of interacting particles per unit cell [21], the mass of the particles [22], the intensity of cross-correlation between noises [23], etc.

In some of our earlier papers on the subject [18,24,25] we have considered a correlation ratchet in which Brownian particles are subjected to both a thermal equilibrium noise and a symmetric three-level telegraph process (trichotomous noise). Addressing the case in which the telegraph noise is very flat, we have shown that the transport direction of Brownian particles can be controlled by thermal noise even if the transport is induced by *symmetric* trichotomous noise. On the basis of the large-flatness limit we have also established a number of cooperation effects. Namely, for certain values of the system parameters there occur more than two CRs versus the switching rate and temperature of noise, and at large spatial asymmetries the

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current exhibits characteristic "disjunct windows" (DWs) with respect to the switching rate and temperature; within DWs the direction of the current is opposite to that in the surroundings. The advantage of such a multinoise model involving thermal noise is that the temperature as the control parameter can be easily varied both in experiments and in potential technological applications.

However, within the framework of the calculation scheme of references [18,24,25], the absolute value of the net current is inversely proportional to a large value of the flatness parameter of trichotomous noise, hence the absolute value of the current is very small (infinitesimal). So, the results of references [24,25] are mainly of theoretical interest, while applications are possible at moderate (finite) values of flatness. Thus papers [24,25] leave open the fundamental question, both from the theoretical and practical viewpoints, whether the phenomena of DWs and four CRs versus the switching rate of noise can also occur in the case of finite values of the flatness parameter. The question is addressed in the present paper and the answer is affirmative, which is a crucial result allowing, in practice, to build up effective separation mechanisms of nanoparticles and, in theory, interrelating the phenomena of DWs and multiple (more than two) CRs.

So in the present paper we considerably generalize the model used in reference [18], enlarging the dimension of the system parameter space by one. Namely, in our calculations we allow the flatness parameter to take any values (including moderate ones)and a new interpretation of the results in terms of cross-correlation between two dichotomous noises is presented. Making the model more general and intricate enables us to understand the interrelationship of some effects which formerly stood apart and thereby reveal some new features. Thus we consider one-dimensional overdamped dynamical systems in which Brownian particles move on a spatially periodic piecewise linear asymmetric potential which has one minimum per period. The applied force consists of an additive noise term composed of a thermal noise and of a trichotomous noise. The trichotomous process is a symmetric three-level stationary telegraph process characterized by three parameters: amplitude, correlation time, and flatness [18,26].

One of the main purposes of this paper is to provide an exact formula for the analytic treatment of the dependence of the stationary current on various system parameters (viz., temperature, potential asymmetry, correlation time, flatness, and noise amplitude). We shall show that at moderate values of the flatness parameter there indeed exist certain ranges of flatness values where the phenomena of DWs and four CRs versus the switching rate occur and are interrelated. We shall also derive the necessary conditions for the existence of these effects. It is remarkable that in the region of the parameter space where the DWs exist at moderate flatness the maximal value of the current occurs. Moreover, contrary to the case of large flatness where the DWs are possible only at very large asymmetries of the sawtooth-like potential profile, in the cases considered in the present paper such strong restriction to the potential profile is absent.

Actually, in our earlier papers on the subject we were able to calculate the particle current and other relevant quantities only in certain relative units and thus to provide only a qualitative description of the behaviour of the system. The present generalized model allows us treat moderate flatness and thus to calculate the concrete numerical values of the relevant quantities. For example, as we will see below, for particles of radius 10^{-8} m (e.g. kinesins) in water at room temperature for the ratchet system of the spatial period 100 times greater than the particle radius it follows that in the region of the DWs at moderate flatness the maximal mean value of particles' velocity is in order of magnitude 10 μm/s.

The structure of the paper is as follows. In Section 2 we present the model investigated in this work. A master equation description of the model is given, the corresponding exact stationary solution is discussed, and an exact formula for the current is found and provided. In Section 3 we study the phenomena of DWs and four CRs: analyze the behavior of the current, give the sufficient conditions for the existence of DWs and 4CRs, and demonstrate that the phenomena of DWs and 4CRs are interrelated. This section provides also a new interpretation of the results in terms of cross-correlation between two dichotomous noises. Section 4 contains concluding remarks; also possible applications are discussed. In Appendix a short review of the behavior of the current at different limits (such as the slow-noise limit, fast-noise limit) and low-temperature case, is presented.

2 Model and the exact solution

We consider overdamped motion of Brownian particles in the one-dimensional spatially periodic potential $\tilde{V} = \tilde{V}(\tilde{x} + L)$ with period L and barrier height $\tilde{V}_0 = \tilde{V}_{max} - \tilde{V}_{min}$. The additional force consists of the thermal noise $\tilde{\xi}(\tilde{t})$ with temperature T, and the colored three-level Markovian noise $\tilde{Z}(\tilde{t})$. The system is described by the stochastic differential equation

$$\kappa \frac{d\tilde{X}}{d\tilde{t}} = -\frac{d\tilde{V}(\tilde{X})}{d\tilde{X}} + \tilde{\xi}(\tilde{t}) + \tilde{Z}(\tilde{t}), \tag{1}$$

where κ is the friction coefficient. The thermal fluctuations $\tilde{\xi}(\tilde{t})$ are modeled by the zero-mean Gaussian white noise with the correlation function $\langle \tilde{\xi}(\tilde{t}_1)\tilde{\xi}(\tilde{t}_2)\rangle = 2\kappa k_B T \delta(\tilde{t}_1 - \tilde{t}_2)$, where k_B is the Boltzmann constant. Regarding the random function $\tilde{Z}(\tilde{t})$, we assume it to be a zero-mean trichotomous Markovian stochastic process [18,26] which consists of jumps between the three values $\tilde{z}_1 = -\tilde{a}$, $\tilde{z}_2 = 0$, $\tilde{z}_3 = \tilde{a}$. The jumps follow one another in time according to a Poisson process, while the three values occur with the stationary probabilities $P_s(\tilde{z}_1) = P_s(\tilde{z}_3) = q$ and $P_s(\tilde{z}_2) = 1 - 2q$. In the stationary case the fluctuation process satisfies $\langle \tilde{Z}(\tilde{t})\rangle = 0$ and $\langle \tilde{Z}(\tilde{t}_1)\tilde{Z}(\tilde{t}_2)\rangle = 2q\tilde{a}^2 \exp(-\tilde{\nu} \mid \tilde{t}_1 - \tilde{t}_2 \mid)$, where the switching rate $\tilde{\nu}$ is the reciprocal of the noise correlation time

$\tilde{\tau}_c = 1/\tilde{\nu}$. The probabilities $W_n(\tilde{t})$ that $\tilde{Z}(\tilde{t})$ is in the state n at the time $\tilde{t}$ evolve according to the master equation

$$\frac{d}{d\tilde{t}} W_n(\tilde{t}) = \tilde{\nu} \sum_{m=1}^{3} \tilde{U}_{nm} W_m(\tilde{t}), \tag{2}$$

where

$$\tilde{\mathbf{U}} = \begin{pmatrix} q-1 & q & q \\ 1-2q & -2q & 1-2q \\ q & q & q-1 \end{pmatrix}.$$

The trichotomous stochastic process is a particular case of the kangaroo process [14] with flatness parameter $\varphi = \langle Z^4(t)\rangle / \langle Z^2(t)\rangle^2 = 1/(2q)$.

By applying a scaling of the following form:

$$X = \frac{\tilde{X}}{L}, \;\; V(x) = \frac{\tilde{V}(\tilde{x})}{\tilde{V}_0}, \;\; t = \frac{\tilde{t}}{t_0}, \;\; \xi = \frac{L\tilde{\xi}}{\tilde{V}_0}, \;\; Z = \frac{L\tilde{Z}}{\tilde{V}_0}, \tag{3}$$

we obtain a dimensionless formulation of the dynamics in the potential V with the property $V(x) = V(x-1)$. Choosing $t_0 = \kappa L^2/\tilde{V}_0$, the dimensionless friction coefficient turns to unity and the quantities determining the rescaled noises reduce to

$$\nu = \frac{\kappa L^2 \tilde{\nu}}{\tilde{V}_0}, \quad a = \frac{\tilde{a} L}{\tilde{V}_0}, \quad D = \frac{k_B T}{\tilde{V}_0}, \tag{4}$$

where $2D$ is the strength of the rescaled zero-mean Gaussian white noise $\xi(t)$. For brevity, in what follows we shall call D simply temperature. The dimensionless dynamics is described by the differential equation

$$\frac{dX}{dt} = h(X) + \xi(t) + Z(t), \quad h(x) \equiv -\frac{dV(x)}{dx}. \tag{5}$$

The two-dimensional process $\{x(t), z(t)\}$ is Markovian and its joint probability densities $P_n(x,t)$ for the position variable $x(t)$ and the fluctuation variable $z(t)$ obey the master equation of the form

$$\frac{\partial}{\partial t} P_n(x,t) = -\frac{\partial}{\partial x} j_n(x,t) - \nu \left[P_n(x,t) - P_s(z_n) \sum_{m=1}^{3} P_m(x,t) \right], \tag{6}$$

where $n = 1,2,3$, and

$$j_n(x,t) = [h(x) + z_n] P_n(x,t) - D \frac{\partial}{\partial x} P_n(x,t) \tag{7}$$

are the current densities in the states (x, z_n).

The stationary current J is then evaluated via the current densities

$$J = \sum_{n=1}^{3} j_n^s(x), \quad j_n^s(x) = \left(h(x) + z_n - D\frac{\partial}{\partial x} \right) P_n^s(x), \tag{8}$$

where $P_n^s(x)$ are the stationary probability densities for the states (x, z_n). It follows from equation (6) that the current J is constant. To derive an exact formula for J, we perform an analysis of the system of equation (6) using a piecewise linear sawtooth-like potential:

$$V(x) = \begin{cases} -(x-d)/d, & x \in (0,d) \bmod 1, \\ \;\;(x-d)/(1-d), & x \in (d,1) \bmod 1, \end{cases} \tag{9}$$

with $d \in (0, 1/2)$.

The $h(x)$ being periodic, the stationary distributions $P_n^s(x)$ as solutions of the system of equation (6) are also periodic and it suffices to consider the problem in the interval $[0,1)$. The force corresponding to the sawtooth potential (9) is

$$h(x) = \begin{cases} h_0 := 1/d, & x \in (0,d), \\ h_1 := -1/(1-d), & x \in (d,1). \end{cases} \tag{10}$$

As the force $h(x)$ is piece-wisely constant, equation (6) splits up into two linear differential equations with constant coefficients for the two vector functions $\mathbf{P}_i^s(x) = (P_{1i}^s, P_{2i}^s, P_{3i}^s)$, $i = 0,1$, defined on the intervals $(0,d)$ and $(d,1)$, respectively. The solution reads as

$$P_{ni}^s(x) = P_s(z_n) \left[\frac{J}{h_i} + \sum_{k=1}^{5} C_{ik} A_{nik} \exp\left(\frac{\lambda_{ik} x}{D} \right) \right], \tag{11}$$

where $n = 1,2,3$, $i = 0,1$, $k = 1,\ldots,5$, C_{ik} are constants of integration,

$$A_{nik} = [\lambda_{ik}(h_i + z_n - \lambda_{ik}) + \nu D]^{-1}, \tag{12}$$

and $\{\lambda_{ik}, k = 1,\ldots,5\}$ is the set of roots of the algebraic equation

$$\lambda_i^5 - 3h_i\lambda_i^4 + (3h_i^2 - a^2 - 2\nu D)\lambda_i^3 + (4D\nu + a^2 - h_i^2) h_i \lambda_i^2 + \nu D(\nu D - 2h_i^2 + 2qa^2)\lambda_i - D^2\nu^2 h_i = 0. \tag{13}$$

Ten independent conditions, for the ten constants of integration C_{ik} and the current J, can be established by requiring continuity and periodicity of the quantities $\mathbf{P}_i^s(x)$ and $j_{ni}^s(x)$ at the points of discontinuities, that is,

$$P_{n0}^s(d) = P_{n1}^s(d), \quad P_{n0}^s(0) = P_{n1}^s(1),$$
$$j_{n0}^s(d) = j_{n1}^s(d), \quad j_{n0}^s(0) = j_{n1}^s(1), \quad n = 1,2,3. \tag{14}$$

As it follows from equation (6) that $J = \text{const}$, the system of linear algebraic equations (14) contains only ten linearly independent equations. By including the following eleventh equation (i.e., normalization condition):

$$\sum_{n=1}^{3} \int_0^1 P_n^s(x) dx = 1, \tag{15}$$

a complete set of conditions is obtained for determining the ten constants of integration C_{ik} and the current J.

This procedure leads to an inhomogeneous set of eleven linear algebraic equations. Now the current J can be expressed as a quotient of two determinants of the eleventh degree:

$$J = \frac{\det[B_{lr}(1-\delta_{r,1}) + \delta_{l,11}\delta_{r,1}]}{\det(B_{lr})}, \tag{16}$$

where the matrix (B_{lr}) (with $l, r = 1, \ldots, 11$) is defined as follows:

$$\begin{aligned}
&B_{n+3\,1} = B_{n\,1} = \frac{1}{h_0} - \frac{1}{h_1},\ B_{n+6\,1} = B_{10\,1} = 0,\\
&B_{11\,1} = \frac{d}{h_0} + \frac{1-d}{h_1},\ B_{n\,k+5i+1} = A_{nik}\exp\left(\frac{\lambda_{ik}d}{D}\right),\\
&B_{n+3\,k+5i+1} = A_{nik}\exp\left(\frac{\lambda_{ik}}{D}\delta_{i,1}\right),\\
&B_{m+6\,k+5i+1} = (h_i - \lambda_{ik})B_{m\,k+5i+1},\\
&B_{m+8\,k+5i+1} = (h_i - \lambda_{ik})B_{m+3\,k+5i+1},\\
&B_{11\,k+5i+1} = \frac{D}{\lambda_{ik}}\sum_{n=1}^{3} P_s(z_n)(B_{n\,k+5i+1} - B_{n+3\,k+5i+1}),
\end{aligned} \tag{17}$$

with $n = 1, 2, 3$, $m = 1, 2$, $k = 1, \ldots, 5$, $i = 0, 1$. Here $\delta_{l,r}$ is the Kronecker symbol, and the quantities A_{nik} and λ_{ik} are the same as in equation (11).

3 Results: four current reversals and disjunct "windows"

The quantity of central interest to us is the average particle current J and its response to the switching rate ν. Figure 1 exhibits the level curves of zero current, $J(D, \nu; d, a, q) = 0$, for fixed d and q at various values of $a = \text{const}$. The level curves may be considered as functions $D = D(\nu)$, with a, d, and q being parameters. In Figures 1a and 1b the level curves at the top close to the larger finite values of temperature D (not shown), whereas in all the panels both branches of the level curves on the right approach zero as ν grows. Regarding the zeros of the function $D(\nu)$: if $a < a_0(q, d)$ (see Eq. (23) in Appendix), then D becomes zero only at the limit $\nu \to \infty$; if $q < 0.25$ and $a_1(q, d) > a > a_0(q, d)$, (see Eq. (24) in Appendix), then D approaches zero at two finite values of the switching rate ν; whereas if $a > a_1(q, d)$, there is only one zero at a finite value of ν. In view of this, two types of level curves are distinguishable in Figure 1: namely, the connected ones (e.g., curves 2 and 3 in Fig. 1a) and the ones comprising two components, viz., a closed curve and a curve whose one end is open (e.g., curve 1 in Fig. 1a). There is one very special level curve (e.g., curve 5 in Fig. 1b) which intersects itself at the saddle point. Let us note that in the case of Figure 1c both branches of the level curves on the right approach zero only at the limit $\nu \to \infty$.

Using Figure 1 as a visual aid, we will now examine DWs in the region of noise parameters, where $q < 0.25$ and $a < a_0(q, d)$. As mentioned above, the current exhibits characteristic disjunct zones of temperature and switching rate on the borders of which CRs occur. That is, for certain values of (d, a, q) there exist closed curves in the plane $(D, \nu; D \neq 0)$ on which CRs take place (see Fig. 1). The closed curves encircle the regions where the direction of the current is negative, whereas in the surrounding regions the current direction is positive (see also Fig. 2).

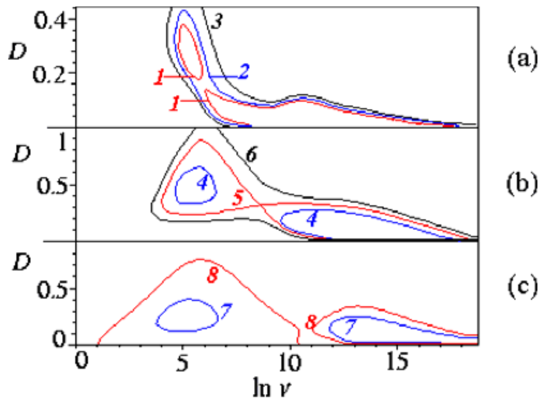

Fig. 1. The level curves $J(D, \nu) = 0$ at $d = 0.007$. The noise amplitude a and the transition probability q have the following values: (1) $a = 22$; (2) $a = 23$; (3) $a = 26$; (4) $a = 89.5$; (5) $a = 103.5$; (6) $a = 117.5$; (7) $a = 177$; (8) $a = 235$; and (a) $q = 0.0003$, (b) $q = 0.1$, (c) $q = 0.2$.

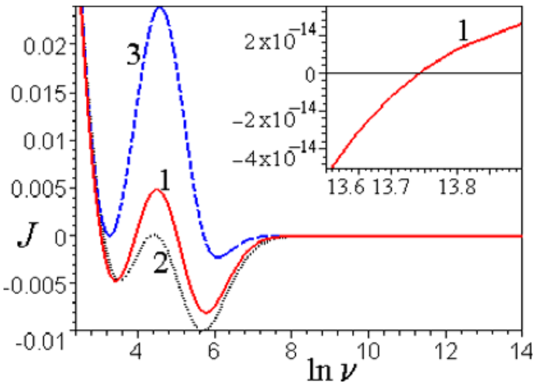

Fig. 2. Four current reversals vs. the switching rate ν. For all the curves $d = 0.1$, $q = 0.159$, and $a = 13.1$. Solid line (1): $D = 0.081$. Dotted line (2): $D = 0.10147$. Dashed line (3): $D = 0.04807$. The current $J = J(\nu)$ is computed by means of the exact equation (16). Note the occurrence of the phenomenon of 4CRs in the case of curve 1. The inset depicts curve 1 in the region of the fourth current reversal.

The DWs exist if the surface $J(D, a, \nu) = 0$ (with fixed parameters $d = \text{const.}$ and $q = \text{const.}$) has a local extremum at the noise amplitude $a_{ex}(q, d)$, a saddle point at the noise amplitude $a_s(q, d)$, and $a < a_0(q, d)$ (see also Fig. 3 and Eq. (23) in Appendix). By varying a, d, and q step by step, we will obtain all the values of d, q, and a in case of which the DWs exist: to every point within the shaded regions in Figure 4 corresponds one closed curve on which $J(D, \nu) = 0$. For fixed q, the region of the DWs shrinks to a critical four-point C, which is characterized by the coordinates: $d_c(q)$, $a_c(q)$, $D_c(q)$, and $\nu_c(q)$. The values $d_c(q)$ and $a_c(q)$ are respectively the upper and lower values of the parameters for the DWs to occur, i.e., the

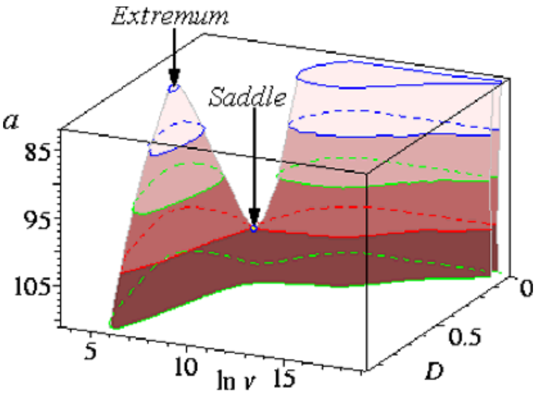

Fig. 3. The surface of current reversals $J(D, a, \nu) = 0$ at $d = 0.007$ and $q = 0.1$. The coordinates of the extremum point are $a = 82.5$, $D = 0.46$, and $\ln \nu = 8.5$, whereas those of the saddle point are $a = 103.5$, $D = 0.30$, and $\ln \nu = 5.5$.

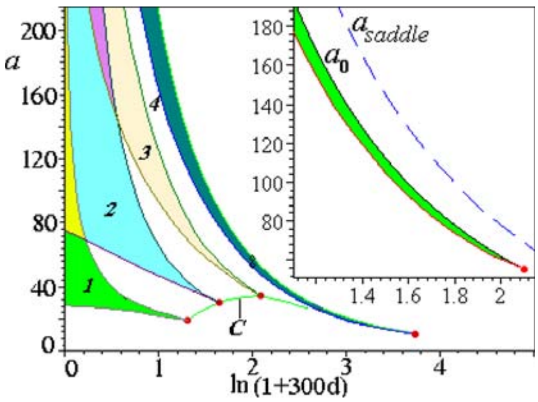

Fig. 4. The necessary and sufficient condition for the existence of the DWs. The transition probability has the following values: (1) $q \to 0$; (2) $q = 0.05$; (3) $q = 0.10$; and (4) $q = 0.168$. The inset presents the necessary and sufficient condition for the existence of the DWs for the value of the transition probability $q = 0.2$.

phenomenon is possible if $d \in (0, d_c)$ and $a \in (a_c, \infty)$ (see Fig. 4). Let us emphasize that the critical parameters d_c and a_c depend only on the parameter q describing the flatness of nonequilibrium noise.

In Figure 5 we have plotted the critical parameters d_c and a_c as functions of the flatness parameter φ, $q = 1/(2\varphi)$. For increasing values of q the critical amplitude $a_c(q)$ starts from the value $a_c(0) \approx 19.4$, and grows to the local maximum $a_{c\,max} \approx 39.6$ at $q \approx 0.084$. Next it decreases, attaining the local minimum $a_{c\,min} \approx 4.4$ at $q \approx 0.178$, and then approaches the value $a_c(0.25) \approx 230$ as $q \to 0.25$. The critical asymmetry parameter $d_c(q)$ exhibits a bell-shaped form as q is varied: the parameter d_c increases from 0.009 to 0.5 if q grows from zero to 0.178, and then decreases to $d_c(0.25) \approx 0.007$ as $q \to 0.25$. The sufficient conditions for the existence of DWs effect at different values of the parameter q are shown in Figure 4 by the shaded wedge-shaped areas in the plane (a, d). The lower border of the shaded region is determined by the curve $a = a_{\rm ex}(q, d)$ with fixed q. In the context of

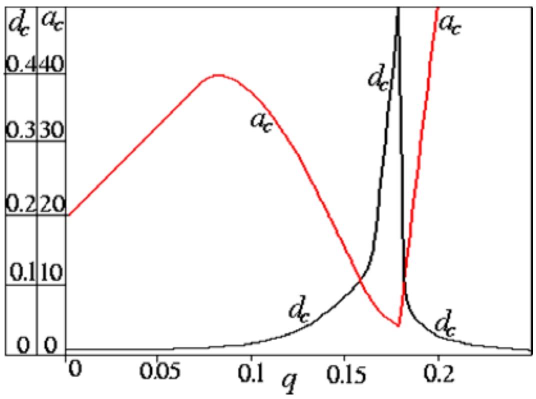

Fig. 5. The critical values of the parameters for the DWs effect to occur: a_c and d_c vs. q. The DWs phenomenon is possible if $d \in (0, d_c)$ and $a \in (a_c, \infty)$.

the upper borders the following characteristic regions can be discerned for noise-flatness:

(i) if $q < 0.178$, the upper border consists of two lines. In the neighborhood of the critical point C the upper border is determined by $a = a_s(q, d)$ whereas if $d < d_0$, d_0 is the solution of the equation $a_s(q, d_0) = a_0(q, d_0)$, that is given by $a = a_0(q, d)$. At the critical point C the saddle and extremum points merge;
(ii) in the case of $0.25 > q > 0.178$, the values of the amplitude $a_s(q, d)$ are always greater than $a_0(q, d)$, i.e., $a_s(q, d) > a_0(q, d)$. Hence, in this case the upper border is given by the curve $a = a_0(q, d)$ with fixed q (cf. also Fig. 1c).

Let us note that DWs can also appear for $q > 0.25$ as well as for $a > a_0(q, d)$, but if $q > 0.25$ the phenomenon of 4CRs is absent. An important observation here is that the phenomenon of DWs will occur only if the noise-flatness φ is greater than 1.695.

Turning to the four-current-reversal effect as a function of the switching rate ν (see Fig. 2), the effect occurs only if $q < 1/4$. In a general case, depending on the values of noise-flatness, the four-current-reversals can be realized in two ways. In the case of large flatness, $q < 0.015$, the four CRs effect exists if the upper branch of the function $D = D(\nu)$, which describes the level curve in Figure 1, has a local minimum (e.g., the curve (1) in Fig. 1a). For moderate flatness the phenomenon of four CRs is associated with DWs [see Figs. 1b and 1c]. Figure 6a shows a phase diagram in the (q, a) plane at $D = 0.08$ and $d = 0.002$. To the regime where four CRs occur at fixed values of d and D in the figure correspond two disconnected regions: the large boomerang-like shaded arc and the small comma-like stroke/dash at the horizontal coordinate axis. (In our earlier papers on the subject [24,25] we observed the 4CRs effect only in the infinitesimal parts of the small comma-like regions.) In the large boomerang-like shaded region the current reversals are associated with the DWs effect. Notably, as a rule, in the finite interval of the flatness parameter the 4CRs effect is absent [e.g., $0.0122 < q < 0.0238$ in Fig. 6a]. An interesting circumstance about the phase

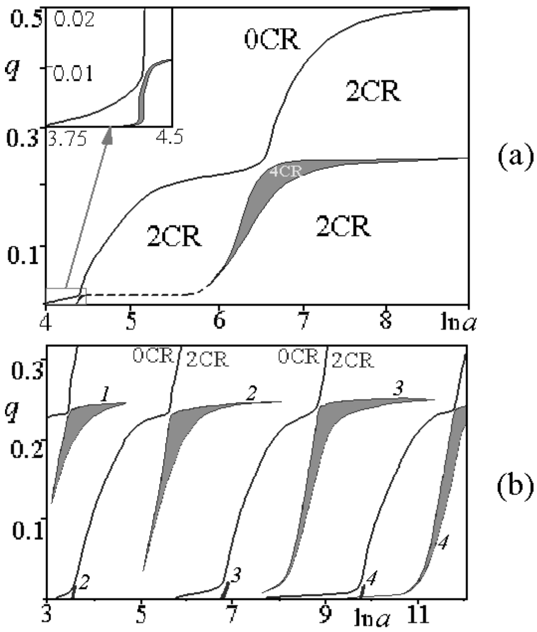

Fig. 6. The necessary and sufficient condition for the existence of the 4CRs effect. The asymmetry parameter and temperature have the following values: (a) $d = 0.002$, $D = 0.08$; (b): (1) $d = 0.050$, $D = 0.1014$; (2) $d = 0.005$, $D = 0.0955$; (3) $d = 0.0002$, $D = 0.0931$; (4) $d = 0.00001$, $D = 0.09268$.

diagrams in Figure 6 is that the characteristic temperature D^*, at which the upper "bottle-neck" between the regions with no current reversals and with 4CRs disappears, is a slowly varying function of the asymmetry parameter d. For example, in the interval $0.05 > d > 10^{-5}$, the temperature D^* grows monotonically from $D = 0.0927$ to $D = 0.1014$ as the asymmetry parameter increases [see Fig. 6b]. The tendency that is apparent in Figure 6b, namely, a decrease in the flatness interval between the domains with 4CRs as the asymmetry of the potential grows, is also valid for lower asymmetries ($d > 0.005$). An important observation here is that the growth of the potential asymmetry will increase the region of the phase space (q, a) where the 4CRs with DWs effect appear.

It is amusing to note that in a very narrow region of the parameter space an extra interaction takes place and the effect of six current reversals versus the switching rate occurs (see Fig. 7). In this region the boomerang-like formation 4 intersects the comma-like stroke 4 (see the right-hand bottom of Fig. 6b). The region is determined by the following values of the parameters: $d \approx 0.00001$, $q \approx 0.00066$, $a \approx 17240$, and $D \approx 0.0927$.

It is interesting that the results of the present paper can be interpreted in terms of cross-correlation intensity between two dichotomous noises. Namely, the trichotomous noise $Z(t)$ in equation (5) can be presented as the sum of two cross-correlated zero-mean symmetric dichotomous noises $Z_1(t)$ and $Z_2(t)$, i.e., $Z(t) = Z_1(t) + Z_2(t)$.

The dichotomous noises $Z_1(t)$ and $Z_2(t)$ are characterized as follows: $z_1, z_2 \in \{\frac{1}{2}a, -\frac{1}{2}a\}$ with $\nu_1 = \nu_2 = \nu$ and

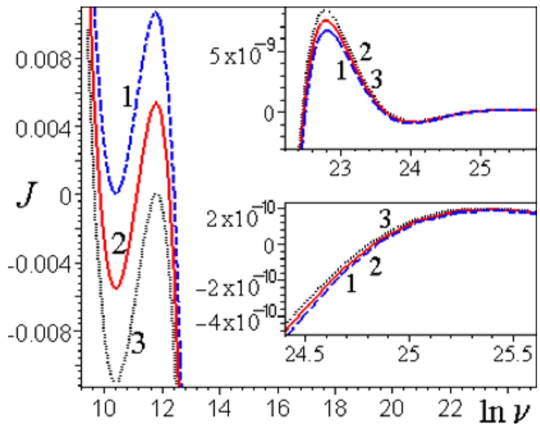

Fig. 7. Six current reversals vs. the switching rate ν. For all the curves $d = 0.00001$, $q = 0.00066$, and $a = 17240$. Dashed line (1): $D = 0.09258$. Solid line (2): $D = 0.09268$. Dotted line (3): 0.09278. The absolute minima (not shown in the figure) of all the three curves occur within the range $14.35 < \ln \nu < 14.40$, whereas $-0.121 < J < -0.119$. Note that the scaling is different along the axes of the two insets.

the correlation function

$$\langle Z_i(t')Z_j(t)\rangle = \rho_{ij}\frac{a^2}{4}e^{-\nu|t-t'|}, \quad i, j = 1, 2, \qquad (18)$$

where $\rho_{ii} = 1$, and $\rho_{ij} = \rho \in (-1, 1)$ with $i \neq j$ is the cross-correlation intensity of the noises Z_1 and Z_2. In this case the probability $q = (1 + \rho)/4$, from which it follows that the correlation coefficient ρ and the flatness φ of the trichotomous noise $Z(t)$ must be related as $\rho = (2 - \varphi)/\varphi$. It is obvious that the critical noise-flatness $\varphi = 2$ corresponds to $\rho = 0$, i.e., to the case of two statistically independent dichotomous noises. Hence the necessary condition for the existence of four CRs with DWs considered in the present paper is that the correlation intensity ρ is negative. Let us note that such a cross-correlation between dichotomous noises may result from the following two reasons: the two noises are either partly of the same origin or are influenced by the same factors. Notably, the phenomenon of cross-correlation-induced current reversals has earlier been considered in the context of qualitatively different ratchet models with multiplicative and additive noises [23].

4 Discussion and concluding remarks

We have provided the exact formula (16) for the current J of overdamped Brownian particles in a sawtooth ratchet potential subjected to both thermal noise and additive zero-mean trichotomous noise. A major virtue of the proposed model is that an interplay of symmetric three-level colored and thermal noises in ratchets with simple asymmetric sawtooth potentials can generate a rich variety of cooperation effects, e.g., multiple current reversals (more than two) versus noise correlation time and disjunct "windows" of temperature and switching rate where the direction of the current is opposite to that in the surroundings.

The major results of the present paper establish the effects of both 4CRs and DWs at moderate values of noise-flatness, (see also Figs. 1 and 2). We emphasize that both phenomena occur only if the flatness parameter $\varphi > 1$. Thus, in the case of the symmetric dichotomous noise ($\varphi = 1$) the effects of DWs and 4CRs are not possible. Notably, as a rule, for moderate values of flatness ($50 > \varphi > 2$), the phenomenon of four current reversals is related to the effect of disjunct "windows".

Though we are not aware of any simple physical explanation for the above-mentioned effects, the distinct behaviors of the currents induced by dichotomous and trichotomous noises are not surprising if we remember that there is the so-called flashing barrier effect at $\varphi > 1$, which generates a counter current induced by dichotomous noise. An excellent explanation for the flashing barrier effect can be found in reference [13].

The sufficient conditions for the existence of DWs and 4CRs at different values of the flatness parameter are shown in Figures 4 and 6, respectively. Two circumstances should be pointed out: (i) there is the lower limit of noise-flatness, namely $\varphi = 2$, beyond which the phenomenon of 4CRs disappears; (ii) for a fixed value of the flatness parameter, there are the upper value $d_c(\varphi)$ of the potential asymmetry parameter d and the lower value $a_c(\varphi)$ of the noise amplitude a for the DWs to occur, i.e., the phenomenon is possible if $d < d_c(\varphi)$ and $a > a_c(\varphi)$. The critical parameters d_c and a_c depend only on the flatness of noise.

It is remarkable that in the case of large flatness, $\varphi > 50$, the DWs appear only if the asymmetry of the potential profile is very large, $d < 0.009$, whereas in the case of moderate flatness, such restriction weakens considerably. For example, if $\varphi = 2.8$, then the potential profile can be nearly symmetric, $d < 0.49$.

It is obvious that in the DWs the intensity and the direction of the current J can be controlled by thermal noise (see also Fig. 1). Moreover, as in equation (4), the friction coefficient κ is absorbed into the time scale, so in the original (unscaled) set-up, the particles of different friction coefficients are controlled by different switching rates. According to the suggestions in references [2,7,9,10], this can lead to an efficient mechanism for the separation of different types of particles by exploiting the sensitive dependence of the current reversals on the switching rate. The possible usefulness of the phenomena of four CRs and DWs was discussed in our earlier papers [24,25]. Comparing two possible techniques of particle separation, one with two CRs (with no use of DWs) [2] and the other with DWs — one can see certain advantages of the latter. In the case of two current reversals, the zeros of the current $J(\nu)$ generally occur at more greatly displaced values of ν than in the case of DWs with 4 CRs. The last feature of DWs seems to be applicable and useful for the particle separation schemes with high selectivity within a narrow ν interval [24], because it enables one to obtain a sharp negative extremum of $J(\nu)$ with a relatively large absolute value. Although also in the case of usual 2CRs the wide interval of ν between zeros of J can be made narrower by varying the remaining system parameters, the really narrow intervals occur only in the vicinity of the transition regimes (i.e., the transition from two current reversals to zero current reversals), where the absolute value of the current is very small. Another advantage of the model with DWs is that the control parameter is temperature, which can easily be varied in experiments.

Let us note that in the unscaled set-up (3) the mean value of the velocity of particles can be computed from the equation

$$\langle v \rangle = \frac{k_B T}{\kappa L} \cdot \frac{J}{D}, \tag{19}$$

where the quantities in the first fraction are dimensional, whereas in the second one dimensionless.

As an example, we consider particles of radius 10^{-8} m (e.g. kinesins) in water. At temperature $T = 310$ K one obtains from the Stokes' formula the value of the friction coefficient $\kappa \approx 2\times 10^{-10}$ kg/s. Assuming a diffusion regime $\tilde{V}_0 \sim 10 k_B T$, the spatial period of the ratchet system $L = 10$ μm and[1] $J/D \approx -0.5$, we infer that $\langle v \rangle = -11$ μm/s or otherwise $\langle v \rangle \approx -4$ cm/h.

We examined the phenomena of four current reversals and disjunct "windows" in models similar to equation (1) already in reference [24]. However, in contrast to the present paper, our earlier paper [24] considered only the case where the value of the of the flatness parameter of the trichotomous noise was very large (infinite), and hence the absolute value of the current very small (infinitesimal).

What concerns the interpretation of the results in terms of cross-correlation intensity between two noises, it is important to note that the necessary condition for the existence of four CRs with DWs considered in the present paper is that the correlation intensity ρ is negative.

The effect of four CRs seems to suggest that cross-correlation between colored noises may provide some understanding as to why structurally very similar motor proteins with two heads, such as kinesin and dynein motor families, move in opposite direction on the micro-tubules while being in the same environment and experiencing the same periodicity, e.g. the conventional kinesin and ncd [27]. It remains to be seen whether the current reversals due to the effects of cross-correlation between noises may play a role in cell biology by considering the motion of motor proteins.

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Appendix: Asymptotic regimes

The asymptotic regimes of the current J have been in greater detail discussed in references [15,18]. Here we will outline only the points needed in this paper.

[1] For moderate flatness the typical values of J_{min}/D, in the region where the DWs occur, are within the interval $(-0.05, -1)$.

First we will briefly review the asymptotic limits of the current J as a function of ν and D. We will begin with considering the dependence of the current J on the switching rate ν. In the case of the adiabatic limit, $\nu \to 0$, the current is positive and changes with temperature as follows: if trichotomous fluctuations induce transitions back and forth over the potential barrier, i.e., if $a > h_0$, the current J decreases monotonically as the temperature rises. On the other hand, if trichotomous transitions do not induce transitions in both directions over the barrier, i.e., if $a < h_0$, the net current exhibits a bell-shaped extremum. Hence, there is an optimal temperature maximizing the current.

In the fast-noise limit, $\nu \to \infty$, the current can be expressed as

$$J \approx \frac{qa^2 h_0 |h_1| (h_0^2 - h_1^2) e^{1/D}}{2\nu^{5/2} D^{7/2} (e^{1/D} - 1)^2}, \quad D \neq 0, \tag{20}$$

so that the current is positive and decays algebraically to zero in $\nu^{-5/2}$. It can be easily seen that the functional dependence of the current on the temperature D is of a resonance-like form. The optimal value of the temperature that maximizes the current is equal to 0.309. It is noteworthy that the limits $D \to 0$ and $\nu \to \infty$ do not commute: the formulas for the current J will be different, depending on which order the limits are approached, either $D \to 0$ and $\nu \to \infty$, or $\nu \to \infty$ and $D \to 0$. In the absence of thermal noise, the current decays to zero as $J \sim \pm \exp(-C\nu)$ with a positive constant C (see also Refs. [15,18]).

Thus, if $D \neq 0$, at the asymptotic limits of both small and large ν the function $J = J(\nu)$ is always positive. Hence, there can exist either none or an even number of current reversals versus ν.

Next we will consider the dependence of the current J on the temperature D. At high temperatures, $D \to \infty$, one can show that J is positive and decays algebraically to zero in D^{-4} at a rate proportional to a^2, or more precisely,

$$J \approx \frac{q(h_0 + h_1)a^2}{180 h_0 |h_1| D^4}. \tag{21}$$

The zero-temperature case has been discussed in reference [15].

At low temperatures, $D \to 0$, in the case $a < h_0$, the current is positive for all values of q, ν, and d. However, in the case $a > h_0$, trichotomous fluctuations can induce the current reversals if certain additional conditions are met. Remarkably, J is positive at any value of ν if $a \leq |h_1| + h_0$. If $a > |h_1| + h_0$, the current reverses to negative at critical values of the switching rate $\nu = \nu_0(q, a, d)$. For numerical calculations of the current reversal points $\nu_{\rm cr}$ at zero temperature, $D = 0$, $J(\nu_{\rm cr}) = 0$, the following transcendental equation can be used [15]:

$$\begin{aligned}\gamma_1\{\gamma_0[\alpha_0(a) + \alpha_0(-a)] + qa[\alpha_0(-a) - \alpha_0(a)]\} = \\ \gamma_0\{\gamma_1[\alpha_1(a) + \alpha_1(-a)] + qa[\alpha_1(-a) - \alpha_1(a)]\},\end{aligned} \tag{22}$$

where

$$\gamma_i = \sqrt{(1-2q)h_i^2 + q^2a^2},$$

$$\alpha_i(a) = \exp\left[-\frac{\nu}{h_i^2(a^2 - h_i^2)}(qa^2 - h_i^2 + a\gamma_i)\right] - 1, \quad i = 0, 1.$$

At the low temperature limit, $D \to 0$, the behavior of the function $J = J(D)$ is not uniform. Depending on the values of the remaining system parameters, at $D = 0$ the function $J = J(D)$ may have a (finite) positive, a (finite) negative, or a zero value and may start to increase or decrease for a while as the temperature D increases [18,25]. What is important about the asymptotic limits in the context of the present paper, where we are interested in the behavior of the current in the intermediate domains of the system parameters, is the fact that by changing the temperature, we have to distinguish between the following two cases. First, if

$$a^2 < a_0^2 \approx \frac{2qh_0^2}{1-2q}\left\{1 + \frac{3}{2}\left[\exp\left(\frac{2\sqrt{2}|h_1|}{3h_0}\right) - 1\right]\right\} + (h_0 + |h_1|)^2, \tag{23}$$

or if $a > a_0$ and $\nu < \nu_{\rm cr\,1}$ or $\nu > \nu_{\rm cr\,2}$, there can occur either an even number of CRs versus D or none; here the critical switching rates $\nu_{\rm cr\,2} > \nu_{\rm cr\,1}$ are the solutions of equation (22). Second, if $a > a_0$ and $\nu_{\rm cr\,1} < \nu < \nu_{\rm cr\,2}$, there always occur an odd number of CRs. Notably, at $q < 1/4$ and

$$a^2 > a_1^2 = \frac{(1-2q)(h_0^2 + h_1^2)}{1-4q} \times \left[1 + \sqrt{1 - \frac{(h_0^2 - h_1^2)^2(1-4q)}{(1-2q)^2(h_0^2+h_1^2)^2}}\right] > a_0^2 \tag{24}$$

the critical switching rate $\nu_{\rm cr\,2}$ tends to infinity at zero temperature.

For the case of the large flatness parameter $\varphi = 1/(2q) \gg 1$, an exhaustive analysis of the phenomenon of the current reversals is presented in references [18,24,25].

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